Dynamic Automated Market Making

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Abstract

Automated market-making (AMM), initially introduced by Bancor and Uniswap, has recently gained extreme adoption, thanks to DEFI and liquidity mining's notable growth. However, there is still room for further development. Several proposals were made to improve various aspects of AMM, including reducing capital requirement [1], preventing front-running [2], and mitigating impermanent loss [3]. This paper proposes a novel approach called Dynamic Automated Market Making (DMM) to allow both flexible fee adjustment and dynamic pricing curve setups. DMM helps reduce the impact of impermanent loss as a result of its flexible fee approach while allowing better capital efficiency using different pricing curve setups tailored particularly according to the tokens pairs in the pool.

1 Introduction

Market making is a well-known financial market activity, including creating orders on both sell and buy sides for a particular asset, which has a primary goal is to make the asset liquid. Market makers and (i.e. the market participants who do market making) are able to earn profits via the trading spread (difference between buy and sell prices). Traditionally, market makers are

large trading firms or brokerage houses that require massive capital requirements and technical know-how, not to mention the licensing requirement.

The rise of decentralized trading and DeFi adoption in cryptocurrency markets has promoted a brand new way of market-making, called Automated Market-Making (AMM). Empowered by smart contracts, AMM performs on-chain pricing deterministically for one crypto token against another based solely on their current inventory. Most AMMs offer the ability for anyone to passively be a part of the market-making process by pooling their inventory together. By contributing to the AMM inventory, users become liquidity providers (LP). The pool smart contract automatically registers each LP's participation, along with their fair share in the pool at any moment, and ensures the correctness of activities such as contributions and withdrawals.

With lucrative incentive scheme via liquidity mining [1], AMMs gained excellent participation from retail users. The most popular ones include Uniswap [4], Curve [5], Balancer [6], Bancor [7], and Mooniswap [2]. While most of them offer similar typical features, including liquidity pooling and pricing based on a determined formula (and thus can be automated by smart contracts), each has different trade-offs and benefits. Uniswap is arguably the purest form of AMM with a simple constant-product design. Meanwhile, Curve focuses on swapping between a pair of similar assets (e.g. between stablecoins or wrapped tokens of the same underlying assets). Balancer offers dynamic configurations of the assets in a pool (e.g. more than two assets, various asset values ratios). Other recent proposals, including Dodo [3] and Mooniswap [2] provide different solutions to reduce impermanent loss. That said, the AMM design space is still in its early phases. Solutions to address existing issues, including capital efficiency, front-running and impermanent loss are still yet to be explored.

1.1   Motivation & Observation.

Our observation is that the AMM space can be inspired by the professional market-making space, which has been well studied and optimized over

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[1]https://hackernoon.com/with-yield-farming-liquidity-mining-and-games-defi-truly-puts-your-tokens-to-work-kk2k348f

decades. Most AMMs currently are based on one-size-fit-all models [2]. In other words, regardless of the price stability between the assets in the pool, they have the same price curving and fee structure. For example, a stablecoin pair (e.g. USDC/USDT) works poorly on standard constant-product AMMs, since the fixed price curve is built to cater towards all types of pairs [3]. To the best of our knowledge, Curve is the first solution that offers specific pricing curves with customised configurations for several similar-asset pairs (including stablecoin pairs), but their solution is applicable to only a limited number of asset pairs.

In addition, current AMM price spreads (i.e. difference between bid/ask prices) are also market neutral (only dependent on pool inventory), which is quite counter-intuitive. In a traditional capital market, price spreads should also be dependent on market conditions (e.g. wider when market is volatile and thinner when market is stable).

Our goal is to bring the flexibility and the fine-grained control level that professional market making has to AMM space. We particularly focus on two different areas, namely dynamic pricing curves that tailored specifically to each pair and flexible fee model that is self-adjust based on market conditions, to offer more competitive features to AMMs.

1.1.1 Measuring Price Stability Between Pairs

Given that the key determinant of potential for capital efficiency and risk of impermanent loss is the relative price stability between the pool's assets, there is room for improvements in capital efficiency and negations of impermanent loss by using different price curves and fee models for different pairs.

In order to quantify the price stability between a pair of tokens X and Y, we propose three measuring metrics below.

- `Ratio:` the price of token X in terms of token Y at a given point in time.

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[2]Balancer is arguably more flexible by allowing various asset combination within a pool.
[3]The slippage will be the same for both stablecoin pairs and a new Token/ETH pair.

- **Amplitude:** the max ratio divided by the min ratio across a given period. A higher amplitude represents a bigger shift in the max price ratio between the two assets.

- $\sigma_{\mathbf{mean}}$: the standard deviation of the mean of the ratios. The higher this deviation is, the greater the volatility between the ratios.

Amplitude is considered the best indicator for the tokens pair's impermanent loss and price movement out of range. Meanwhile, the standard deviation indicates how much relative price moves. Table 1 and Figure 1 outline several examples of token pairs and their amplitude metrics over 365 days.

Token Pair	Amplitude	Amplitude (exld. 1% outliers)	σ_{mean}	Price Stability
AMPL / ETH	29.131	22.207	0.716	Weak/No Correlation
WBTC / ETH	2.24	2.109	0.181	Correlated
EURS/USDC	1.268	1.227	0.05	Strongly Correlated
USDT/USDC	1.037	1.022	0.003	Similar Assets

Table 1: Examples of a few token pairs and their corresponding 3 metrics over a period of 365 days

The lower the amplitude and variance, the higher potential there is for high levels of capital efficiency and low impermanent loss. Based on this key idea of price stability, we believe we can create a generalized model that programmatically allows for different pricing curves and fee models for different asset pairs.

1.2   Results

First, we introduce a novel LP fees model to take into account the market condition in the price spread for AMM. This new LP fee model arguably attracts more trades during less-volatile market conditions, and charges higher fees (thus higher LP revenue) during volatile ones. Second, we address the capital efficiency in AMM (i.e. better capital utilisation to offer better liquidity) by leveraging additional information regarding the price stability between the tokens in the pool. Our final protocol, called Dynamic Automated

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Marketing Making, collectively offers various pool configurations for AMM depending on assets in the pool and allows AMM to reflect better with the market changes.

2 Background

Most AMMs do not require a price oracle. Instead, the price is solely based on the two tokens' inventories. This principle is implemented by almost all notable AMMs in DeFi like Uniswap, Balancer, Curve, Kyber's APR, Bancor and Mooniswap.

In this paper, we define X, Y as two token assets in a pool, and x, y as their respective current inventories. A trade will be either to sell X to buy Y, resulting in an increase in x and decrease in y, or vice versa when a user sells Y to buy X. X's price against Y depends on the bonding curve, which describes the relation between X and Y in Cartesian coordinate axis. This curve can be formulated as a bijective and strictly decreasing function

(a) ampl/eth pair

(b) wbtc/eth pair

(c) eurs/usdc pair

(d) usdt/usdc pair
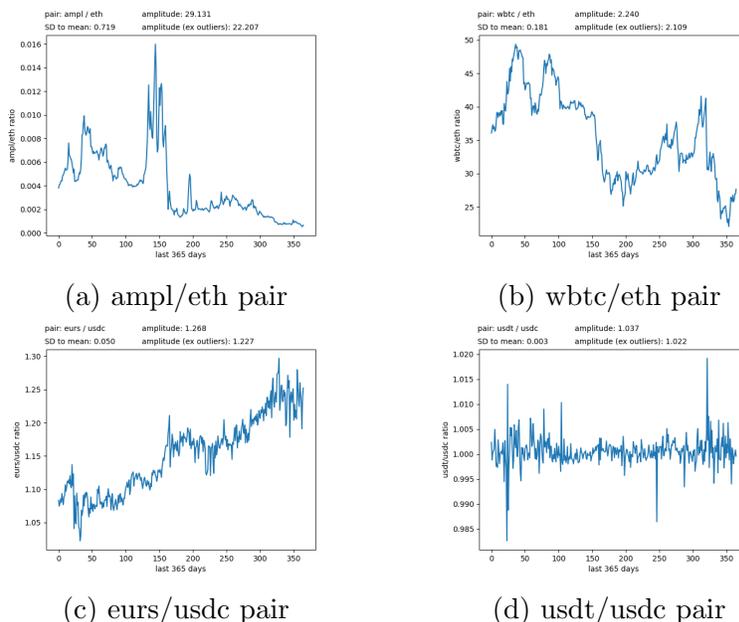
Figure 1: Amplitude for various token pairs in the last 365 days

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$f : R+ \rightarrow R+$, which respects the following properties:

- $\forall x \in R+, \exists! y \in R+$ that, $y = f(x)$ (bijective)

- $\forall y \in R+, \exists! x \in R+$ that, $y = f(x)$ (bijective)

- $\forall x_1, x_2$, if $x_1 > x_2$ then $f(x_1) < f(x_2)$ (strictly decreasing)

The following sections examine popular XY curves in the existing DeFi projects.

2.1 Uniswap curves

Uniswap is based on the constant-product model, which means $x \cdot y = k$, where k is constant. In other words, the price of y is determined as $y = \frac{k}{x}$. This symmetric curve is illustrated in Figure 2. The curve has been well explained in many other sources and used in production for a long time. Several projects are using Uniswap-like curves, such as Bancor v1 [7] and Mooniswap [2].
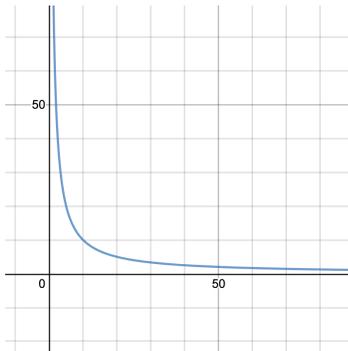
Figure 2: Uniswap curve
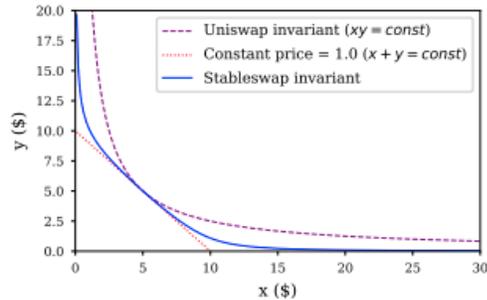
Figure 3: Stableswap curve

2.2 Curve: Stableswap

Stableswap Curve [5] focuses on stablecoin pairs or similar asset pairs (i.e., wBTC/renBTC). Stableswap Curve includes a variant of a constant-sum

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curve for a wide range of inventory movement called "constant-sum range" while the rest of the inventory ranges behaves similarly to a constant-product curve (Figure 3). The advantage of a constant sum curve is its small slippage rate in the target range. However, this approach is only optimized for similar token pairs pools, like stablecoin pairs or synthetic / wrapped token pairs. Stableswap Curve cannot be used for arbitrary pair of assets, especially for price discovery use cases.

2.3   Balancer curves

Balancer [6] is also based on a constant-product model, but it adds a weight for each asset in the inventory function. Basically, the inventory function can be represented as $y = \frac{k}{x^n}$ where k is a constant and $n > 0$. Balancer's bonding curve is illustrated in Figure 4 with $n = 1.5$. Their main motivation is to allow various combinations of asset's values distribution in the pool instead of fixing it at $1 : 1$ ratio like on Uniswap.
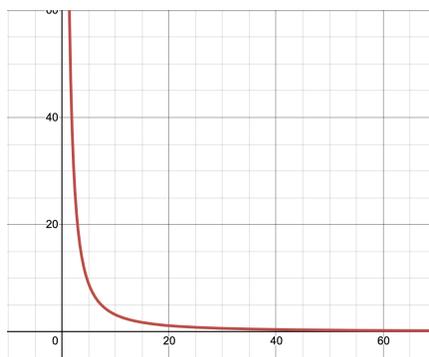
Figure 4: Balancer curve

2.4   Comparison between different solutions

Table 2 shows a comparison between different market-making solutions. Kyber APR was introduced back in 2018 by the Kyber team to improve the capital utilisation for AMM, but it was not adopted widely due to the lack of liquidity pooling support [1].

	Uniswap	Curve	Balancer	Dodo	Kyber APR	Kyber DMM
Impermanent loss	Yes	No	Yes	Less	Yes	Varied
Price oracle	No	No	No	Yes	No	No
Liquidity pooling	Yes	Yes	Yes	Yes	No	Yes
Capital efficiency	Low	High	Low	High	High	High
Stable coins optimization	No	Yes	No	No	No	Yes
Gas consumption	Low	High	High	High	High	Low

Table 2: Comparison between different market making solutions

3   DMM: Dynamic Market Making

In this section we discuss 2 mechanisms — amplification of the price curve to boost capital efficiency and dynamic fees based on market volatility to potentially mitigate impermanent losses and increase LP profit.

3.1   Dynamic pricing curves

Dynamic pricing curves try to improve capital efficiency of the Uniswap model. The curves are still a constant product, but of virtual balances instead of real balances. Thanks to the virtual balances, which are amplified significantly from real balances, the DMM pools can achieve moderate spread and slippage rates compared to the Uniswap model given the same capital.

3.1.1   Technical details

We define liquidity providers' initial contribution to the pool as x_0 and y_0, and $x_0 \cdot y_0 = k$. The initial virtual balances are defined as $x_0' = x_0 \cdot a$ and $y_0' = y_0 \cdot a$, in which a **is the amplification factor** and $a > 1$.

In the beginning, the real and virtual balances of the two assets are $(x, y) = (x_0, y_0)$, $(x', y') = (x_0', y_0')$ respectively.

Instead of using the simple constant-product function $x \cdot y = k$, the pool with **Dynamic pricing curve model** will maintain a constant product in virtual balances by using the new inventory function:

$$x' \cdot y' = k'$$

The constant k' can be derived from k as follows:

$$x' \cdot y' = k'$$
$$\Rightarrow x_0' \cdot y_0' = k'$$
$$\Rightarrow (x_0 \cdot a) \cdot (y_0 \cdot a) = k' \tag{1}$$
$$\Rightarrow k' = k \cdot a^2$$
$$\Rightarrow x' \cdot y' = k \cdot a^2$$

The pool using the DMM model maintains both virtual balances and real balances. After some trades, the real balances and virtual balances are updated to be:

Real balances:

$$\begin{cases} x = x_0 + \Delta_x \\ y = y_0 + \Delta_y \end{cases}$$

Virtual balances:

$$\begin{cases} x' = a \cdot x_0 + \Delta_x \\ y' = a \cdot y_0 + \Delta_y \end{cases}$$

Let P be the price function of X against Y, P_0 the initial price, $P_0 = \frac{y_0}{x_0}$, and P_{min}, P_{max} the minimal and maximal price supported by the Dynamic pricing curve respectively. The pool will run out of token X or Y when real balances are zero:

1. If $x_0 + \Delta_x = 0$, $\Delta_x = -x_0$. The virtual balances are:

$$\begin{cases} x'_{min} = x_0 \cdot a - x_0 \\ y'_{max} = \frac{x_0 \cdot y_0 \cdot a^2}{x_0 \cdot a - x_0} \end{cases}$$

Hence, $P_{max} = \frac{y'_{max}}{x'_{min}} = \frac{x_0 \cdot y_0 \cdot a^2}{(x_0 \cdot a - x_0)^2} = \frac{y_0}{x_0} \cdot \left(\frac{a}{a-1}\right)^2$

We set $\frac{y_0}{x_0} = P_0$, so $P_{max} = P_0 \cdot \left(\frac{a}{a-1}\right)^2$

2. If $y_0 + \Delta_y = 0$, so $\Delta_y = -y_0$. The virtual balances are:

$$\begin{cases} y'_{min} = y_0 \cdot a - y_0 \\ x'_{max} = \frac{x_0 \cdot y_0 \cdot a^2}{y_0 \cdot a - y_0} \end{cases}$$

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$$P_{min} = \frac{y'_{min}}{x'_{max}} = \frac{(y_0 \cdot a - y_0)^2}{x_0 \cdot y_0 \cdot a^2} = \frac{y_0}{x_0} \cdot \left(\frac{a-1}{a}\right)^2, \text{ so } P_{min} = P_0 \cdot \left(\frac{a-1}{a}\right)^2$$

In summary, we see that users benefit from lower spread and slippage when the pools use the new pricing curve. However, this comes at the expense of the price range no longer being unbounded, but being restricted between $P_{min} = P_0 \cdot \left(\frac{a-1}{a}\right)^2$ and $P_{max} = P_0 \cdot \left(\frac{a}{a-1}\right)^2$.

For example, when $a = 2$, the virtual balances are double the real balances in the original constant-product model. The price range support for this is from $\frac{P_0}{4}$ to $4P_0$. The inventory curves of Uniswap, Curve and Dynamic pricing curve are visualized in Figure 5.
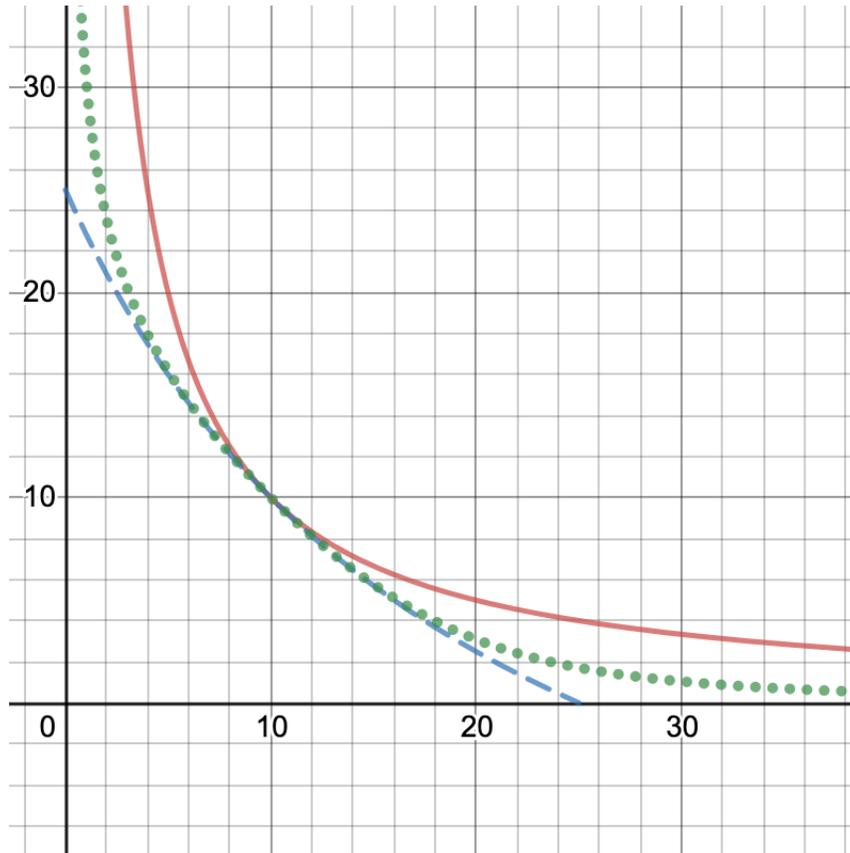
Figure 5: Inventory curves of Uniswap (red), Curve (green) and Dynamic pricing curve (blue)

3.2 Dynamic Fees

Besides the dynamic pricing curve model, we also introduce a dynamic LP fee model that adjusts the AMM LP fee based on market condition. Most existing AMMs have a simple LP fee model (e.g. a fixed percentage of the trade value) regardless of the market condition. This does not reflect what is happening in traditional capital market, in which market makers adjust the trading spread based on market condition to either protect themselves (when market is fast-moving) or attract more trades to incentivize more trading. We wanted to bring this important fee/market volatility correlation to AMMs.

Particularly, in our dynamic fee model, the LP fee is increased when market is moving fast (i.e. becoming volatile), and is reduced when the market is stable (less volatile). In a quiet market, the dynamic fee model encourages the user to trade more by offering tighter spreads through the reducing of the LP fees. The increased volume attracted by the lower spread will arguably make up for the revenue loss due to the reduction of the LP fees. On the other hand, our dynamic fee AMM will increase its spreads by charging higher LP fees when the market is volatile. Traders or users who trade in such volatile markets will receive fewer tokens than average, as the profit is kept in the pool to reduce potential impermanent loss for LPs. Thus, this dynamic fee model encourages market stability in Defi through spread adjustments.

There are several ways to implement this dynamic fee adjustment mechanism deterministically. In this paper, we propose one particular approach that is based fully on on-chain information, which are determined by on-chain volume of each pool. This information is publicly available and can be efficiently recorded and updated for an AMM pool after every trade. We describe how to implement this approach in the rest of this Section.

3.2.1 Dynamic Fee Adjustment based on Volume

In this section, the definition of the bonding function in section 2 is reused. Based on this bonding function, when users want to sell Δ_x, the formula for Δ_y they get in the Dynamic Fee model is:

$$\Delta_y = f(x) - f(x + \Delta_x \cdot (1 - fee - z))$$

In which:

- x, y are the current inventories of 2 assets X, Y

- fee is the base fee, pre-defined by AMM

- z is the variant factor, dependent on the average volume of AMM during a time period. It must satisfy the condition $0 < 1 - fee - z \leq 1$, alternatively written as $-fee \leq z < 1 - fee$

To determine the fluctuation of the AMM's volume, the AMM compares its volume between the short window and long window.

We denote A_s to be the average volume of AMM in the short window and A_l to be the average AMM volume in the long window. The ratio between the volume of these two windows can be defined as $r = A_s/A_l$. Thus, $0 \leq r < \infty$. A variant factor z will be represented as a function of parameter r.

For example, z can be a Sigmoid function:

$$z(r) = \frac{m \cdot (r - n)}{\sqrt{p + (r - n)^2}}$$

In which m, n, p are chosen to satisfy the condition $-fee < z < 1 - fee$ and $m > 0$. n is the break point to encourage or discourage trades. If $r > n$, then $z(r) > 0$, the trades in dynamic fee model are more expensive than in non-dynamic fee models, thus discouraging trades. On the other hand, if $r < n$, then $z(r) < 0$, causing trades in the dynamic fee model to be more beneficial.

As an example, in a case where $fee = 3bps = 0.003$, we choose $m = 0.009, n = 1, p = 8$. When AMM has no volume: $r = 0$, $z = -0.03$, so $1 - fee - z = 1$, dynamic fee model benefits traders with zero fee and zero spread. When r is very big, z is close to its peak at 0.009.

The implementation of r and z can be deeply customized. For example, r can be calculated using Simple Moving Average (SMA) or Exponential Moving Average (EMA), etc. Each application may design its own $z(r)$ function to suit its objective.

4 Implementation Examples For Optimized Pools

In Observations, we explained how the price stability of a token pair is the key determinant of potential for capital efficiency, as well as risks of impermanent loss (IL), and highlighted 4 broad categories of token pairs. We also outlined 2 programmable mechanisms - dynamic pricing curves and dynamic fees.

4.1 Examples

We suggest implementations where, depending on the likely stability of price between the pairs in the pool, we can leverage 3 parameters/mechanisms to provide higher capital efficiency and potentially mitigate impermanent losses by shifting profits from takers to the LPs.

1. Amplification Factor

2. Base Fees

3. Dynamic Fees

In general, we believe the higher the stability of prices between the pairs, the higher the amplification can be, the lower the base fees need to be and the dynamic fees lower. Conversely, the more volatile the stability of prices between the pairs, the lower the amplification amount we can apply to the pool, and the more important it is to use base fees and dynamic fees to reduce IL risk.

We use the 4 categories outlined in Observations to explain how a generalized model could be applied to optimize various types of asset pairs. For a few pairs in each category, we calculate suitable amplification factors based on data of the past one year, and suggest base / dynamic fee parameters that would be appropriate.

4.1.1 Similar Assets Pairs

These include stablecoin pairs (USDC-USDT, DAI-USDC) and similar token pairs (WBTC-renBTC).

Pair	Amplitude (ex. outliers)	Amplification (ex. outliers)	Amplitude (fr. mean)	Amplification (fr. mean)
USDT - USDC	1.022	93	1.012	186
DAI - USDC	1.042	49	1.027	73
WBTC - renBTC	1.019	109	1.012	139

Here, we can see that for relatively stable pairs, we will be able to leverage amplification factors of well above 100, assuming the pool was started at the mean. For assets pairs that are meant to be similar, but have a higher level of variance (for example DAI-USDC), we can leverage a lower amplification factor.

For example, with an amplification of 100, a pool of 1M USDT and 1M USDC will be able to achieve a spread and slippage equal to a Uniswap pool of 100M USDT and 100M USDC, essentially achieving a capital efficiency gain of about 100X. With that amplification, the price ratio between the pair can fluctuate around [0.98 - 1.02] of the initial price ratio.

The tradeoff for pools with high amplification levels, while highly capital efficient, is that the price can go out of range, and the pool will likely be inactive in the period when there is a price mismatch with the wider market. Asset pricing moving out of range is much less of an issue in similar pairs, since we've observed market prices quickly recovers back into the 1:1 ratio.

Possible Base Fee	Possible Fee Range
3bps	2bps - 6bps

For pairs of this category, a base fee of 3bps and a dynamic fee range of [2bps - 6bps] would likely be appropriate. We expect dynamic fees to play the role of encouraging trading activity when volume is low, and helping LPs earn more fees when volume is higher.

The base fees for this type of pool can be very low, given the very low risk of LPs in providing liquidity. At the same time, LPs will be able to facilitate

a far higher trading capacity per unit contributed and hence earn more fees compared to less capital efficient methods.

4.1.2 Strongly Correlated Assets Pairs

These would include forex pairs that tend to move within a given range, with lower amounts of volatility.

Pair	Amplitude (ex. outliers)	Amplification (ex. outliers)	Amplitude (fr. mean)	Amplification (fr .mean)
EURS - USDC	1.226	10	1.109	20
XSGD - USDC	1.164	13	1.086	24

For these types of token pairs, the variance is significantly higher, with a lower probability of the price ratio returning to a previous mean compared to similar asset pairs. If the amplification is too high, the probability of the pool being inactive is much higher.

Based on past data, we can see that amplification of 10 could be applied for this category of pools, where the capital efficiency of the Dynamic pricing curve will be 10 times better than unamplified pools, and the price range is $[0.81 - 1.23]$ of the initial price ratio.

Possible Base Fee	Possible Fee Range
10bps	6.7bps - 20bps

A base fee of 10bps and a dynamic fee range of [6.7bps - 20bps] would likely be appropriate. Given that impermanent loss risk for forex pairs is low yet not negligible, the slightly higher fees generated during periods of high volatility should help to mitigate losses.

4.1.3 Correlated Assets

These would include major token pairs, where price movements tend to be correlated.

Pair	Amplitude (ex. outliers)	Amplification (ex. outliers)	Amplitude (fr. mean)	Amplification (.mean)
WBTC - ETH	2.109	3	1.537	5
LINK - ETH	3.296	2	1.873	3

We see here that amplification factors of 2-3 can be applied to pools of this category. An amplification of 3 will allow for a price range of [0.44, 2.25] of the starting price index. It is unlikely to be optimal to create an aggressive amplification factor here, given the high risk of the resulting pool being inactive. It is however likely that an LP might want to do so in order to capture as much fees as possible even with a lower level of liquidity.

Possible Base Fee	Possible Fee Range
20bps	13.7bps - 40bps

Risk of impermanent loss here is high - and therefore, by using a base fee of 20bps and applying the dynamic fee model, we see that there is an increase of 20bps during high trading volumes, allowing the LPs to regain a higher amount of the potential losses in fees.

4.1.4 Weakly/Uncorrelated Assets

These include most token pairs, whose value have no correlation to common quote tokens, such as ETH or USDC.

Pair	Amplitude (ex. outliers)	Amplification (ex. outliers)	Amplitude (fr. mean)	Amplification (fr .mean)
ZRX - ETH	5.23	1.8	2.890	2.5
AMPL - ETH	24.851	1.2	6.806	1.6
CAP - ETH	10.699	1.4	4.989	1.8

Amplification is not suitable for these token pairs due to significant price volatility. We expect no improvement in capital efficiency for token pairs of this nature.

Possible Base Fee	Possible Fee Range
30bps	20bps - 60bps

Risk of impermanent loss here is very high - and therefore, by using a base fee of 30bps and applying the dynamic fee model, we see that there is an increase of 30bps during high trading volumes, allowing the LPs to regain a higher amount of the potential losses in fees.

4.2   Comparison

Here we show how the various example implementations differ in terms of slippage and dynamic fees. The more stable the price between the tokens in the pool, the higher the amplification can be, and the lower the impact of the dynamic fees. Conversely, the less stable the price between the tokens in the pool, the less we can amplify, but the dynamic fees can have a far larger impact.
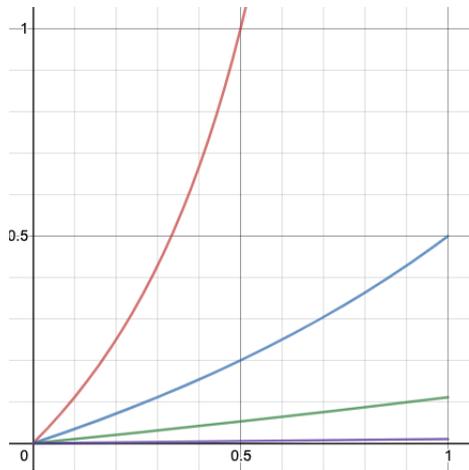
Figure 6: Slippage curves where amplification factor is 100 (purple), 10 (green), 3 (blue), 1 (red)
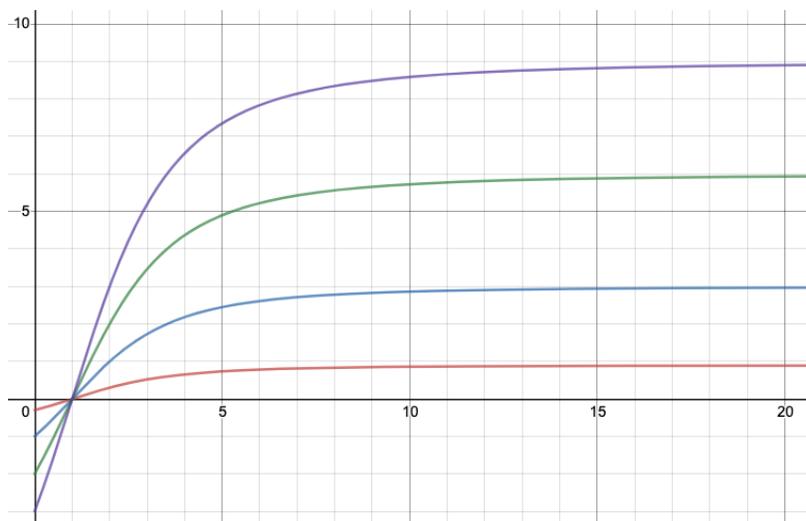
Figure 7: Possible dynamic fee curves where base fee is 3bps (purple),
10bps (green), 20bps (blue) and 30bps (red)

5   Conclusions

In this paper, we show a generalized model for optimizing token pools for potentially improving capital efficiency and mitigating impermanent loss. This model uses 3 simple mechanisms, namely price curve amplification, custom base fees and dynamic fees based on volume to achieve this optimization.

This model is based on 2 key observations — that potential for capital efficiency and risk for impermanent loss is largely dependent on price stability between pairs, and traders are willing to accept larger spread when demand for liquidity is high.

The key benefits of this system is simplicity of implementation, and preserving the original intention of AMMs being unreliant on external data sources or extrinsic incentives. The key tradeoffs include a limited price ratio range, and potentially high data/computational cost for complex gauging of volume trends.

Future work includes dynamic updates of price curves so that all prices are supported, quantifying the impact of dynamic fees on impermanent losses and arriving at effective formulas for estimating volatility and fee adjustments.

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